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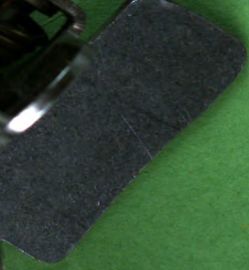
WIDENER



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Charles E.

1837



TESTIMONIAL

RESPECTING THE

PROPORTIONAL FORMULÆ.

NOTE. While this little work was under consideration, another Gentleman, who is also a Professor of the College, was absent from Fredericton, which will account for the appearance of only two signatures.

KING'S COLLEGE, FREDERICTON,
September 12, 1834.

AT the request of Mr. JOHN LEE, we have examined a publication entitled "*Proportional Formulæ*;" and have great pleasure in stating, that the author appears to us to discover an original genius, and to have justly applied the Algebraic principles of Proportion to the object of his publication, namely — the facilitation of that process in Arithmetic, commonly called the Rule of Three."

E. JACOB,
Vice-President.

JAMES SOMERVILLE,
Theological Professor.

Buckingham Palace, Eng.

*With The Compli-
ments Of*

The Author.

August 20, 1830.

**PROPORTIONAL FORMULÆ,
BI-COMPLEX AND TRI-COMPLEX;
OR A
SYSTEM
ALGEBRAICALLY DERIVED FROM THE NATURE OF
PROPORTIONALS,
FOR THE
FACILITATION OF THAT PROCESS IN ARITHMETIC,
COMMONLY CALLED
THE RULE OF THREE.**

BY JOHN LEE,
AUTHOR OF "THEORY OF LONGITUDE."

**CAMBRIDGE PRESS:
METCALF, TORRY, AND BALLOU.
*1837.**

HARVARD UNIVERSITY
WIDENER

*The People of
Jas. Charles V. Buckingham,
of Cambridge,
Rec'd. Nov. 15, 1894.*

Entered according to act of Congress in the year 1837, by **MATCALF, TORRY,
AND BALLOU**, in the Clerk's Office of the District Court of the District of
Massachusetts.

P R E F A C E.

THE Author of these Formulæ has discovered nautical problems, which enable the mariner to obtain, by mere trigonometrical calculation, his precise longitude at sea. The Nautical Almanac is not required, nor any of the data which it contains; and even though utterly unacquainted with his local situation, and ignorant of the *day of the month* and the *hour of the day or night*, he can yet, by these problems, with an azimuth compass and a quadrant, ascertain his latitude and longitude with all the above-mentioned particulars. Their merit and truth have been certified in King's College, Fredericton, New Brunswick, and also in Cambridge, Massachusetts, U. S., by the present Professor of Mathematics in Harvard University. Correspondence has been held respecting them, with the American and British governments. The Author hopes to pre-engage, by a simple statement of these facts, the friendly feelings of the advocates of Science. All such individuals are requested to consider, that, by a candid and liberal encourage-

ment of the present publication, they will probably facilitate the promulgation of more important scientific researches.

In this work as it appeared in New Brunswick, the Tri-complex Formulæ were not given. They are useful, however, to extend and amplify the analysis of the subject, and will frequently furnish available facilities of practical operation.

Cambridge, Mass., U. S.

Sept. 1837.

N. B. The Author of the Proportional Formulæ is prepared to communicate instruction in the Hebrew, Greek, Latin, French, and English languages; and also in Logic and Mathematics.

THE BI-COMPLEX FORMULÆ.

IN performing what is called the Rule of Three, when we have reduced the numbers to their lowest terms, our next object is, to find for these three numbers a fourth proportional — which fourth number is the answer which we want-
ed. Now it is proposed in this treatise to introduce a more brief and easy method for finding this fourth number, than the method now in use; and the fundamental principles of this new method will be easily discerned in the four following formulæ.

$$1. A : (A + m) :: (A + n) : (A + m) + n + \frac{mn}{A}.$$

$$2. A : (A + m) :: (A - n) : (A + m) - n - \frac{mn}{A}.$$

$$3. A : (A - m) :: (A + n) : (A - m) + n - \frac{mn}{A}.$$

$$4. A : (A - m) :: (A - n) : (A - m) - n + \frac{mn}{A}.$$

APPLICATION.

We shall in the first instance conceive the three numbers to be integral, and we shall also observe that these formulæ are most easily ap-

plicable when those numbers are *nearly all equal*. The formulæ when expressed in a verbal rule will be as follows: — Observe the difference of the first and second terms, and place it for convenience, under the third term; also take the difference of the *first* and *third* terms, and place it under the *second* term; now there will be four cases which we shall severally consider; 1st — when both differences are *excesses* over the first term; 2d — when that of the second term is an *excess*, and the other a *defect*; 3d — when that of the second is a *defect*, and the other an *excess*; and 4th — when both are *defects*.

CASE I.

In order to obtain a fraction, take the product of the differences for a numerator, and the first term for a denominator; to either the second or the third term add the difference below it, and state their sum; the obtained fraction being added to that sum gives the true answer.

Example 1.	Example 2.	Example 3.
754 : 757 :: 763 :	831 : 835 :: 832 :	927 : 931 :: 938 :
9 3	1 4	11 4
<hr/>	<hr/>	<hr/>
766 $\frac{27}{754}$.	836 $\frac{4}{831}$.	942 $\frac{44}{927}$.

REMARK. In the present treatise, a fraction is said to be *analyzed*, when it is presented in a form in which it appears evidently composed by the addition of some integral number to a fraction whose value is less than 1. And if the proposed fraction be *itself* less than 1, the added integral number must be conceived to be 0. Thus $\frac{2}{3}$, when analyzed, assumes the form $0 + \frac{2}{3}$.

or $0\frac{3}{4}$, and $\frac{1}{2}$ takes the form $1 + \frac{1}{2}$, or $1\frac{1}{2}$. If the fraction found in Case I. be greater than 1, it is proper to *analyze* it, connecting its integral with the other two numbers, in the prescribed process of addition; the remaining fraction, being now appended to the sum thus found, will give the true answer, as appears by

Example 4.

$$5 : 7 :: 9 :$$

$$\begin{array}{r} 4 \quad 2 \\ 1 \end{array}$$

$$\begin{array}{r} 1 \\ \hline \end{array}$$

$$12\frac{3}{4}.$$

Here the fraction is $\frac{3}{4}$, which when analyzed is $1\frac{3}{4}$.

CASE II.

Subtract from the second term the difference under it, or *add* to the *third* term the difference under it; from the number thus obtained subtract, *in its analyzed form*, the fraction found as before; the result is the true answer.

REMARK. To subtract the analyzed fraction, conceive its integral increased by 1, and when so increased, subtract that integral from the number pre-obtained, and state the remainder; subtract the numerator of the remaining fraction from its denominator, and employ the remainder as the numerator of a *new* fraction having the same denominator as the former one; this new fraction, being annexed to the integral remainder, will give the true answer.

Example 1.

$$356 : 358 :: 351 :$$

$$\quad \quad 5 \quad \quad 2$$

$$\begin{array}{r} \text{Subtract} \quad \overline{353} \\ \quad \quad 0 \frac{10}{358} \\ \hline 352 \frac{14}{358} \end{array}$$

Example 2.

$$273 : 277 :: 271 :$$

$$\quad \quad 2 \quad \quad 4$$

$$\begin{array}{r} \text{Subtract} \quad \overline{275} \\ \quad \quad 0 \frac{2}{273} \\ \hline 274 \frac{28}{273} \end{array}$$

Example 3.

$$54 : 61 :: 45 :$$

$$\quad \quad 9 \quad \quad 7$$

$$\begin{array}{r} \text{Subtract} \quad \overline{52} \\ \quad \quad 1 \frac{9}{54} \\ \hline 50 \frac{11}{54} \end{array}$$

CASE III.

Add to the second term the difference under it, or subtract from the *third* term the difference under it; then proceed as in Case II.

Example 1.

$$576 : 573 :: 578 :$$

$$\quad \quad 2 \quad \quad 3$$

$$\begin{array}{r} \text{Subtract} \quad \overline{575} \\ \quad \quad 0 \frac{3}{576} \\ \hline 574 \frac{17}{576} \end{array}$$

Example 2.

$$829 : 825 :: 831 :$$

$$\quad \quad 2 \quad \quad 4$$

$$\begin{array}{r} \text{Subtract} \quad \overline{827} \\ \quad \quad 0 \frac{2}{829} \\ \hline 826 \frac{28}{829} \end{array}$$

Example 3.

$$473 : 469 :: 477 :$$

$$\quad \quad 4 \quad \quad 4$$

$$\begin{array}{r} \text{Subtract} \quad \overline{473} \\ \quad \quad 0 \frac{14}{473} \\ \hline 472 \frac{11}{473} \end{array}$$

CASE IV.

From either the second or the third term subtract the difference under it; then proceed as in Case I.

Example 1.	Example 2.	Example 3.
435 : 427 :: 413 :	384 : 381 :: 383 :	297 : 293 :: 291 :
22 8	1 3	6 4
<hr/>	<hr/>	<hr/>
405 $\frac{175}{22}$.	380 $\frac{3}{384}$.	287 $\frac{74}{297}$.

AUXILIARY RULES.

RULE 1. Fractional Decimate Approximation.

By this rule we briefly obtain a decimal approximate value for any vulgar fraction whose digits are many. Calling the numerator and denominator the *constituent members* of the fraction, reserve in each member not more than its two or three sinistral digits, dividing those of the numerator by those of the denominator, in the manner of decimal arithmetic; if the number of digits rejected from the numerator be equal to that of those rejected from the denominator, the quotient is the true answer; if it be *greater* by one, two, or more digits, remove the decimal point in the quotient so many digits to the *right* hand; but if *less*, then so many to the *left*; supplying any deficiency of digits in the quotient by ciphers; the result is the true answer.

Example 1.

Given $\left\{ \begin{array}{l} 7856493286 \\ 341236958 \end{array} \right.$

34)785(23·0882, &c. the value required.

Example 2.

Given $\left\{ \begin{array}{l} 3678294326 \\ 859641235137 \end{array} \right.$

859)36(·0419, &c. and by removal of the point, ·00419, &c. the value required.

RULE 2. When any one term contains more digits than another, or when a fraction appears in any term.

If any of the three given terms be encumbered with a fraction, obtain by the preceding rule, its decimal value, and annex that value to the integral of the term; now, if any other term have *no* fraction appended, subjoin to its dextral digit a decimal point, annexing ciphers also, considered as decimal digits, until the *total* number of digits, integral and decimal, be equal in each of the three terms; now rejecting the decimal point from each term, consider *all* the digits as integral, and obtain the answer accordingly, converting the fraction of that answer into a decimal, and adding it to the integral part; now, if, before the rejection of the decimal points, the number of decimal digits in the first term were equal to the *sum* of the numbers of those in the second and third terms, the result is already true; if it were one, two, or more digits *less* than that sum, remove the decimal point in the result so many digits to the *left* hand; but if it were *more* than that sum, remove the point so many digits to the *right*; supplying always by ciphers any deficiency of digits; the result is the true answer.

Example 1.

 $371\frac{1}{4} : 372\frac{1}{10} :: 372\frac{1}{4} :$

or

 $371\cdot4 : 372\cdot1 :: 372\cdot3 :$

by rejecting the point ;

 $3714 : 3721 :: 3723 :$
 $\begin{array}{cc} 9 & 7 \end{array}$

 $3730\frac{63}{100}$

or,

 $3730\cdot016.$

By removal of the point ;

 $373\cdot0016.$

Example 2.

 $28 : 2\frac{1}{2} :: 28\frac{1}{2} :$

or

 $28\cdot0 : 2\cdot83 :: 28\cdot4 :$

by rejecting the point ;

 $280 : 283 :: 284 :$
 $\begin{array}{cc} 4 & 3 \end{array}$

 $287\frac{12}{100}$

or,

 $287\cdot04.$

By removal of the point ;

 $2\cdot8704.$

RULE 3. Simultaneous Multiplication.

In order to multiply any number *at one operation*, by a multiplier consisting of two or more digits ; annex to the multiplicand a decimal point, followed by ciphers, whose number is less by one, than the number of digits in the multiplier ; also, to the sinistral figure of the multiplicand *prefix* the same number of ciphers ; now take the product of the unital digit of the multiplier with the unital digit of the multiplicand ; *again* — take the product of the digit which is immediately *sinistral* of the unital digit in the *multiplier*, with that which is immediately *dextral* of the unital digit in the *multiplicand*, and so on ; employing each time, in the *multiplier*, the digit *sinistral* of the preceding ; and each time, in the multiplicand, the digit which is *dextral* of the preceding, and taking their products as if they were all unital digits ; take the sum of these products for a compound product, setting down its unital digit, and carrying the rest ; now multiply the unital digit of the multiplier into that which is *sinistral* of the

unital in the multiplicand, finding the other products as before, and adding to their sum what was carried; and so proceed.

Example 1.

$$\begin{array}{r} 467 \text{ or } 0467\cdot0 \\ 23 \\ \hline 10741. \end{array}$$

Example 2.

$$\begin{array}{r} 534 \text{ or } 0534\cdot0 \\ 87 \\ \hline 46458. \end{array}$$

REMARK. If the digits of the multiplier be numerous, we may perform several successive operations, removing the result of each operation *sinistrally* from the previous result, as many digits as those which were employed in the production of that previous result. Thus in the annexed example, as the multiplier contains *five* digits, we first multiply by *three* digits, and then by *two*, removing the result of the latter, three digits to the left.

Example.

$$\begin{array}{r} 8793256 \text{ or } 008793256\cdot00 \\ 47359 \\ \hline 3156778904 \\ 413283032 \\ \hline 416439810904. \end{array}$$

RULE 4. The Auxiliary Formulæ.

If $A : B :: C : D$; then $a \cdot A : b \cdot B :: c \cdot C : \frac{b \cdot c}{a} \times D$.

APPLICATION.

If three given terms, as A , B , and C , be severally multiplied by any three numbers taken at

pleasure, as a , b , c , and a fourth proportional be found; that proportional being divided by $\frac{bc}{a}$, will give D , which is the fourth proportional to A , B , and C . Hence, if the differences of A , B , and C be *large*; and if a , b , and c be so selected as to make the differences of $a \cdot A$, $b \cdot B$, and $c \cdot C$, *much smaller*; and if $\frac{bc}{a}$ be either an integral number, or a fraction whose numerator is 1; then the employment of a , b , and c , will facilitate the discovery of D in the following manner. — If the numerical value of $\frac{bc}{a} \times D$ contain a fraction, transform that fraction into a decimal; if $\frac{bc}{a}$ be an integral, divide the numerical value of $\frac{bc}{a} \times D$, by that integral; but if it be a fraction whose numerator is 1, multiply the said value by its denominator; in each case the result is the true answer.

Example 1.

$$\begin{array}{ccc} 456 : 274 :: 913 : \\ 6 \quad 10 \quad 3 \end{array}$$

$$\begin{array}{ccc} 2736 : 2740 :: 2739 : \\ \quad 3 \quad 4 \end{array}$$

$2743\frac{13}{16}$, or decimally 2743.004 . But here $\frac{bc}{a} = \frac{10 \times 3}{6} = 5$; wherefore, dividing the above result by 5, we get for the true answer, 548.6008 .

Example 2.

$$\begin{array}{ccc} 29 : 437 :: 175 : \\ 30 \quad 2 \quad 5 \end{array}$$

$$\begin{array}{ccc} 870 : 874 :: 875 : \\ \quad 5 \quad 4 \end{array}$$

$879\frac{29}{870}$, or decimally 879.02 . But here $\frac{bc}{a} = \frac{1}{3}$; wherefore multiplying by 3, we get for the true answer, 2637.06 .

THE TRI-COMPLEX FORMULÆ.

-
1. $(A + a) : (A + b) :: (A + c) : (A + b) + c$
 $+ \frac{bc}{A + a} - a \times \frac{(A + b) + c}{A + a}.$
 2. $(A + a) : (A + b) :: (A - c) : (A + b) - c$
 $- \frac{bc}{A + a} - a \times \frac{(A + b) - c}{A + a}.$
 3. $(A + a) : (A - b) :: (A + c) : (A - b) + c$
 $- \frac{bc}{A + a} - a \times \frac{(A - b) + c}{A + a}.$
 4. $(A + a) : (A - b) :: (A - c) : (A - b) - c$
 $+ \frac{bc}{A + a} - a \times \frac{(A - b) - c}{A + a}.$
 5. $(A - a) : (A + b) :: (A + c) : (A + b) + c$
 $+ \frac{bc}{A - a} + a \times \frac{(A + b) + c}{A - a}.$
 6. $(A - a) : (A + b) :: (A - c) : (A + b) - c$
 $- \frac{bc}{A - a} + a \times \frac{(A + b) - c}{A - a}.$
 7. $(A - a) : (A - b) :: (A + c) : (A - b) + c$
 $- \frac{bc}{A - a} + a \times \frac{(A - b) + c}{A - a}.$
 8. $(A - a) : (A - b) :: (A - c) : (A - b) - c$
 $+ \frac{bc}{A - a} + a \times \frac{(A - b) - c}{A - a}.$

REMARK. These Formulæ, and also the Bi-complex, could certainly have been given in a much more contracted form, according to the received usages of modern Algebraical notation; but all such modes of expression have been avoided for the following reasons. — The author of this treatise has *invented* a notation which is accurate, simple, and concise, but which, being as yet unrepresented in type, cannot now be introduced. — With respect to the *generalized* use of the received notation, mathematical science has become literally *tainted* by *such* unwarranted license, and the analytic page already teems with symbolized absurdities, which the “operator” changes at will, with all the contemptible dexterity of a juggler. These unreal and fantastic exhibitions of “*nonsense visible*,” being in fact of *no utility*, must be considered merely as an empty and outrageous mockery of the human understanding. The supposition, that absurd premises assist us in discovery, is fallacious. *The shortest Path to Knowledge is from Truth to Truth.* The author intends to consider this subject more largely on a future occasion. At present, however, he has determined, at all inconvenience, to discountenance to the utmost of his power, what he feels to be an unprofitable *debauchery* of science.

APPLICATION OF THE TRI-COMPLEX FORMULÆ.

By the assumption of A at pleasure, determine the values of a , b , and c ; then ascertain the true answer by that formula, to which the case appears to belong.

2*

REMARK. The following notes will facilitate the management of the two fractions which appear in each of these formulæ.

NOTE 1. If to any integral number, a fraction whose numerator is 0 be appended by the additive or subtractive sign, the value of the whole expression is equal to that of the integral. Thus $7 = 7 + \frac{0}{29}$, or $7\frac{0}{29}$; and also $= 7 - \frac{0}{29}$.

NOTE 2. If to any integral number, when diminished by 1, a fraction whose numerator is equal to its denominator be appended by the additive sign, the value of the whole expression is equal to that of the integral before its diminution. The result is also the same, if such a fraction be appended by the *subtractive* sign, to an integral number when *increased* by 1. Thus, $3 = 2 + \frac{1}{7}$, and also $= 4 - \frac{1}{7}$.

NOTE 3. In this treatise a fraction is said to be *demeted*, when it is conceived to be produced by subtracting from some integral number a fraction whose value is less than 1. Thus $\frac{13}{7}$, when demeted, appears in the form $13 - \frac{1}{7}$.

NOTE 4. To present an analyzed fraction in a demeted form, and *vice versâ*. To obtain a new fraction, subtract the present numerator from its denominator. Take the remainder for a new numerator, preserving the former denominator. If the proposed fraction be given in an analyzed form, to its integral when in-

creased by 1 annex the new fraction by the subtractive sign; if otherwise, to its integral when *diminished* by 1, annex it by the *additive* sign; the result is the form required. Thus the analyzed fraction $11\frac{2}{3}$, when demeted, becomes $12 - \frac{1}{3}$, and $7 - \frac{4}{5}$, when analyzed, becomes $6 + \frac{1}{5}$, or $6\frac{1}{5}$.

NOTE 5. In the numerical application of these formulæ, it will frequently be found more convenient to *demete* a fraction than to *analyze* it. Thus, in ascertaining the value of

$$a \times \frac{(A + b) + c}{A + a},$$

if $A = 741$, $a = 6$, $b = 2$, and $c = 3$; if we *analyze*

$$\frac{(A + b) + c}{A + a},$$

it becomes $07\frac{46}{17}$, or simply $7\frac{46}{17}$, which gives

$$a \times \frac{(A + b) + c}{A + a} = 44\frac{76}{17},$$

which, in order to be analyzed or demeted, will require a *new* operation; but if we *demete*

$$\frac{(A + b) + c}{A + a},$$

it becomes $1 - 7\frac{1}{17}$, which *at once* gives

$$a \times \frac{(A + b) + c}{A + a} = 6 - 7\frac{6}{17}.$$

NOTE 6. To obtain the sum of two fractions, when both are *analyzed*, and have the same denominator. Find the sum of the two integral parts. Find also a

new fraction, having the same denominator, and having for its numerator the *sum* of the other two numerators. Having analyzed or demeted this fraction, add its integral to the sum of the other two integral parts, connecting the fractional part by the additive or subtractive sign, by which it was formerly connected. Thus, $(8\frac{1}{2}) + (3\frac{1}{2}) = 11 + \frac{1}{2} = 11 + (1 + \frac{1}{2}) = 12\frac{1}{2}$. Or $(8\frac{1}{2}) + (3\frac{1}{2}) = 11 + \frac{1}{2} = 11 + (2 - \frac{1}{2}) = 13 - \frac{1}{2}$.

NOTE 7. To obtain the same result, when other circumstances are the same, but both fractions are *demeted*. Proceed as before, till the new fraction be obtained; then, either *analyze* that fraction, and subtract its integral part from the sum of the other two integrals, annexing to the remainder the fractional part, by the *subtractive* sign; or else, *demete* that fraction, and, proceeding as before, annex by the *additive* sign. Thus, $(9 - \frac{6}{13}) + (6 - \frac{1}{13}) = 15 - (1\frac{7}{13}) = 14 - \frac{7}{13}$. Or $(9 - \frac{6}{13}) + (6 - \frac{1}{13}) = 15 - (2 - \frac{9}{13}) = 13 + \frac{9}{13}$, or $13\frac{9}{13}$.

NOTE 8. To obtain the same result, &c. when *one* is *analyzed*, and the *other demeted*. Find the sum of the two integrals. Find a new fraction having the same denominator, but having for its numerator the *difference* of the other two. If the analyzed fraction had the greater numerator, subjoin, by the *additive* sign, the new fraction to the sum of the two integrals; but if otherwise, by the *subtractive* sign. Thus, $(9\frac{7}{13}) + (12 - \frac{1}{13}) = 21\frac{6}{13}$. But $(7\frac{3}{16}) + (8 - \frac{1}{16}) = 15 - \frac{9}{16}$.

NOTE 9. To obtain the *difference* of two fractions which are circumstanced as in Note 6. Find the new fraction as in Note 8. If the two integrals be equal, the new fraction is the difference required. If unequal, take their difference. If the greater of the two integrals be connected with the greater numerator in the fractional part, annex the new fraction to the difference of the integrals, by the *additive* sign; if otherwise, by the *subtractive* sign. Thus, the difference of $(13 \frac{3}{10})$ and $(11 \frac{3}{10}) = 2 + \frac{4}{10}$, or $2 \frac{4}{10}$.

NOTE 10. To obtain the difference, when circumstanced as in Note 7. Proceed as in Note 9, only *reversing* the rule which determines the connecting sign. Thus, the difference of $(12 - \frac{8}{10})$ and $(8 - \frac{4}{10}) = 4 - \frac{4}{10}$; and that of $(13 - \frac{3}{10})$ and $(11 - \frac{7}{10}) = 2 \frac{4}{10}$.

NOTE 11. To obtain the difference, when circumstanced as in Note 8. Find the new fraction as in Note 6, and analyze or demete it. If the two integrals be equal, this fraction is the difference required. If otherwise, take the difference of the two integrals, and, by Note 1, or Note 2, give to this difference a form similar to that of the new fraction. If, of the two proposed fractions, that which had the greater integral, were *demeted*, then take, by Note 9, or Note 10, the difference of the new fraction and the quantity last found; the result is the difference required; if otherwise, their *sum*, being found by Note 6, or Note 7, is the difference required. Thus, the difference of $(13 - \frac{7}{11})$ and $(12 \frac{2}{11}) =$ that of 1 and $(1 \frac{4}{11})$, or, by

Note 1, = that of $(1\frac{0}{11})$ and $(1\frac{4}{11})$, which, by Note 9, = $\frac{4}{11}$. But that of $(9\frac{7}{11})$ and $(5 - \frac{7}{11}) = 4 + (1\frac{7}{11})$, which, by Note 6, = $5\frac{7}{11}$.

NOTE 12. By Note 1, and Note 2, an integral number and a fraction may undergo any of the preceding operations. Thus, $(3\frac{5}{8}) + 14 = (3\frac{5}{8}) + (14\frac{0}{8}) = 17\frac{5}{8}$, &c.

Example of a Case in the Tri-Complex Formula.

$$589 : 583 :: 584 :$$

Here, if we take $A = 590$, then $a = 1$, $b = 7$, and $c = 6$; and the first term will be $A - a$, the second, $A - b$, and the third, $A - c$; wherefore, by Formula 8, the answer will be $(A - b) - c + \frac{b c}{A - a} + a \times \frac{(A - b) - c}{A - a}$, = $583 - 6 + \frac{42}{589} + 1 \times \frac{583-6}{589}$, = $577\frac{42}{589} + 0\frac{577}{589}$, = $578\frac{30}{589}$.

But in general the case will be much simplified by selecting A , so that some one of the three quantities, a , b , and c , shall be = 0, in which case one of the two fractions, being also = 0, will disappear. If a become = 0, the *final* fraction disappears, and the formula becomes identical with one of the Bi-complex, which have been already considered. We shall only require, therefore, to illustrate a case where one of the other two quantities, b and c , becomes = 0.

$$589 : 583 :: 584 :$$

Take $A = 583$. Then $a = 6$, $b = 0$, and $c = 1$, and Formulæ 1 and 3 will both apply to the question.

By Formula 1, the answer is $(A + b) + c + \frac{b c}{A + a}$
 $- a \times \frac{(A + b) + c}{A + a} = 583 + 1 + 0 - 6 \times \frac{583 + 1}{589}$
 $= 584 - 6 \times (1 - \frac{6}{589}) = 584 - (6 - \frac{36}{589}) =$
 $578 \frac{36}{589}$. And Formula 3 will give the same result.

REMARK. All the Auxiliary Rules laid down for the Bi-complex Formulæ, will equally apply to the Tri-complex.

END.

NOTICE.

The following works, by the Author of the PROPORTIONAL FORMULÆ, are now in preparation.

I. PRINCIPIA SONIFICA VOCUM.

In this work, all the elementary sounds which are *possible in human language*, are classified and described; and many important errors are corrected, which have obtained promulgation in the works of the most distinguished philological writers, as Johnson, Walker, and Murray. This

work, from its nature, is available as a key to the pronunciation of *every language* whose pronunciation *is not lost*.

II. THE POLYMORPHAL FORMULÆ.

These curious and extraordinary Formulæ were discovered early in October of the present year, 1837, by the Author of the Proportional Formulæ. Though they have no connexion whatever with infinite or infinitesimal quantities or series, they completely destroy the received theory of Equations, and considerably alter the aspect of modern Algebra.

III. SYNTACTICAL FORMULÆ FOR THE ENGLISH LANGUAGE.

In this work the study of *Grammar* is *at last* rendered *intelligible*, — an achievement which undoubtedly marks a *new era* in the history of the science; the Author having redeemed it from the absurdity and pedantry with which it has been copiously embellished heretofore by the literary *toils* of Grammarians. General principles are also given, by which such Formulæ may be easily discovered for *any* language. The Author himself contemplates the future publication of such Formulæ for the Hebrew, Greek, Latin, and French languages.

